

Recitation 7: Laplace transform

Lecturer: Chenlin Gu

Exercise 1. Write down the definition of improper integral and explain it in the $\varepsilon - \delta$ language.

Exercise 2. Determine whether the integral converges or diverges:

1. $\int_0^{\infty} (t^2 + 1)^{-1} dt$;

2. $\int_0^{\infty} t^n e^{-t} dt$;

3. $\int_0^{\infty} \frac{\sin(t)}{t} dt$;

4. $\int_1^{\infty} \frac{1}{\sqrt{t}} dt$.

Exercise 3. Calculate the Laplace transform of following functions:

1. $f(t) = \sin(bt)$;

2. $f(t) = \cos(bt)$;

3. $f(t) = e^{at} \sin(bt)$;

4. $f(t) = e^{at} \cos(bt)$.

Exercise 4. Find the inverse Laplace transform of following functions:

1. $F(s) = \frac{3}{s^2+4}$;

2. $F(s) = \frac{4}{(s-1)^3}$;

3. $F(s) = \frac{2}{s^2+3s-4}$.

Exercise 5. Use Laplace transform to find the solution of initial value problem:

1. $y'' - y' - 6y = 0, \quad y(0) = 1, y'(0) = -1$;

2. $y'' + \omega^2 y = \cos(2t), \quad \omega^2 \neq 4, y(0) = 1, y'(0) = 0$.